Nonparametric Estimation of Expected Shortfall

ON X C N

Io St te niversity

ABSTRACT e e pec ed o f n nc e n y pop e e n n nc n e en nd po e de ed dd y p ope y c n fo e e e con de o nonp e c e pec ed o f e fo dependen n nc o One p e e e of e ce e o e n e e e ne o ed e on of e o c e *M the tic Fin nce* op n o e cc e e on c n e e ed y o n O n y e e e e e ne o n doe no p od ce o e cc e e on of e o f d e en f e e e ne n e o f

1. INTRODUCTION

e e pec ed o f nd e e e pop e e of n nc fo n o po fo o of A zne De en e nd e o c e dd y p ope y n ene nd ence no de e en e e n con de e en o e nd ed nd eco e o e c e e n e n n nc n e en

Le $\{X_t\}_{t=1}^{n}$ e e e of n o po fo o of o e n pe od of e n Le Y_t $-log\ X_{it}/X_{it-1}$ e ne e o e n o o o e e t pe od ppo $\{Y_t\}_{j=1}^{n}$ dependen on y p oce n d on f nc on F en po e e p c o ze o e con dence e e $-p$

$$\nu_p \quad \mathrm{nf}\{y \quad F\ y \geq \quad -p\}$$

e $-p$ q n e of e o d on F e pec e e e of nonp e c ene e q nd pp e fo n y n y n e con e of po fo o oc on

An d n e of e nonp e c od e n ode f ee nd ence ode o nd o d c d y n pec ed o d on n nc n e en p y conce ned c c of p of e o d on o e e d e ene y n e nd ence p opo n p ope p e c o ode deq e fo e p no e e e nonp e c od c n p y n c n o e An e d n e of e nonp e c pp o o de n e of d dependence e d p e n e con e of n nc o e nonp e c e q con de ed n p pe c n cc o d e d dependence e p c y nce e e ec of e dependence on e nce of e e on c n e c e y pe o n e nce fo d e en f e e e e e pp o e e e ec e y e e ceed nce ndependen nd den c y d on o on e p q c y nde e c ed D nd D' cond on Le d e e L nd en nd oo zen An p c dy y Be n nd nc y c y n o nonp e c n e o n n nc e n c n on dependence e en fo e o d e e nd c e e need fo con de n e dependence n n nc e n d ec y e pp o en y e nonp e c e q con de ed n p pe

n p pe e e e o nonp e c e q One d on e ed p e e e of e ce e o de ned y e q ν_p d on n o de c An e e ne e q p opo d n c e p oy e ne o n n e n e on nd e n e n of e e ce e o ope e e ne o n o d e e on o e cc e o ed fo e c of e on y en nd n o e e o n y e e e nce nd e e n e e o of e e ne e q no nece y e n of p e e e e q A e fo e on of e e p e p e e e of e ce e o ood eno nd e e no need q c y o e ne o n y e p n con de n e ne o n e d q e nce n q n e e on fo ndependen e e nd M on nd dependen en nd

[illegible]

[illegible]

2. NONPARAMETRIC ESTIMATORS

[illegible] ν_p [illegible] $Y_{([n(1-p)]+1)}$ [illegible] of ν_p and $Y_{(r)}$ [illegible] of $\{Y_t\}_{t=1}^{n}$ [illegible]

$$\hat{\mu}_p = \frac{\sum_{t=1}^{n} Y_t I(Y_t \geq \nu_p)}{\sum_{t=1}^{n} I(Y_t \geq \nu_p)} = (np)^{-1} \sum_{t=1}^{n} Y_t I(Y_t \geq \nu_p)$$

[illegible] $I(\cdot)$ [illegible]

[illegible] Let K [illegible] $G(t) = \int_t^{\infty} K(u)\,du$ and $G_h(t) = G(t/h)$ [illegible] $S(x) = 1 - F(x)$ [illegible]

$$S_h(z) = n^{-1} \sum_{t=1}^{n} G_h(z - Y_t)$$

[illegible] of ν_p [illegible] $\nu_{p,h}$ [illegible] of $S_h(z) = p$ [illegible][2] [illegible] G_h and $\nu_{p,h}$ [illegible]

$$\hat{\mu}_{p,h} = (np)^{-1} \sum_{t=1}^{n} Y_t G_h(\nu_{p,h} - Y_t)$$

[2] Its statistical properties and how to obtain the standard errors are considered in Chen and Tang (2005). See also Cai (2002) and Fan and Gu (2003) for ke[illegible]

B d on e p o e en of e e ne e $\nu_{p,h}$ o e ν_p e pec ed e e ne e $\mu_{p,h}$ o d p o e e e on cc cy of e n o ed e μ_p Con n o e e foc of e ne c on

3. MAIN RESULTS

e p ope e of e o nonp e c e e e ed n c on e e cond on

Le $\mathcal{F}_k^l$ e e σ e of e en ene ed y $\{Y_t, k \leq t \leq l\}$ fo $l > k$ e α n co c en n od ced y o n

$$\alpha \ k \quad p_{\mathcal{A}\in\mathcal{F}_1^i, \mathcal{B}\in\mathcal{F}_{i+k}^{\infty}} |P \ AB - P \ A P \ B|.$$

e e d e α n f $_{k\to\infty} \alpha \ k$ e dependence de ed y e α n e e e p ed y e ype of n e Do n fo co p en e d on e fo o n cond on e ed n o dy

e e e $\rho \in$, $\alpha \ k \leq C\rho^k$ fo $k \geq$ nd po e con n C

e d on F of Y_t e y con n o p o y den y f con n o cond de e n $\mathcal{B} \ \nu_p$ ne ood of ν_p e o n d on f nc on of $Y_1, Y_{k+1} F_k$ fo $k \geq$ e cond p de e o nded n $\mathcal{B} \ \nu_p \ E \ |Y_t|^{2+\delta} \leq C$ fo e $\delta >$ nd po e con n C

K e c p o y den y n e en cond on $\int_{-1}^{1} u K \ u du$ nd $\int_{-1}^{1} u^2 K \ u du \quad \sigma_K^2 >$ nd K o nded nd L p z con n o de e $\rightarrow \quad n^{3-\beta} \rightarrow \infty$ fo ny $\beta >$ nd n^4 o $^2 n \rightarrow \quad n \rightarrow \infty$

Cond on e n e e e e e c α n ed y ny co on y d n nc e e nc de e A MA A C e c o y nd d on ode o n nce M y nd e e ed e α n fo A C ode nd enon C o e e nd L edo fo d on ode Cond on con n nd d cond on eq e nde y n o ne fo e n nd p o n den e e n en fo e e e n Cond on nd e e one eq ed y e e ne e e

Cond on eq en on e ne Cond on pec e n e fo e nd nc de $O(n^{-1/3})$ e op o de fo e n e on e cond on e co p e cond on po d y e o o n nce Cond on nd co e pond A p on nd f A p on nd c nd of c e e pec e y

Le $\gamma(k)$ $Cov\{(Y_1 - \nu_p)I(Y_1 \geq \nu_p), (Y_{k+1} - \nu_p)I(Y_{k+1} \geq \nu_p)\}$ fo po e n e e k nd

$$\sigma_0^2(p, n) \{Var\{(Y_1 - \nu_p)I(Y_1 \geq \nu_p)\} \sum_{k=1}^{n-1} \gamma(k)\}.$$

A p on nd e D ydo neq y p y $\sigma_0^2(p, n)$ n fo e n nd con e n $n \to \infty$

e e n e n o ed e μ_p p o de po n of efe ence fo e e ne e De on en n e ppend o nde cond on nd nd fo n y po e κ

$$\hat{\mu}_p - \mu_p \quad p^{-1}\{n^{-1}\sum_{i=1}^{n} (Y_t - \nu_p)I(Y_t \geq \nu_p) - p(\mu_p - \nu_p)\} \quad o_p(n^{-3/4+\kappa})$$

B d ype e p n on e d e fo o n eo e d n e p no y of $\hat{\mu}_p$

O M nde cond on $n \to \infty$

$$\sqrt{pn}\,\sigma_0^{-1}(p, n)(\hat{\mu}_p - \mu_p) \xrightarrow{d} N(\ ,\ ).$$

e eo nd c e p nce of $\hat{\mu}_p$ $\sigma_0^2(p, n)/(np)$ e nce of $p^{-1}\{n^{-1}\sum_{i=1}^{n}(Y_t - \nu_p)I(Y_t \geq \nu_p) - p(\mu_p - \nu_p)\}$ e d n o de n e e p n on e dependence n e o n e e e ec d n e p nce o e co nce n $\sigma_0^2(p, n)$ e n e need cc od e dependence n f e nfe ence n e on nd ype e n fo e o f e on e no e e ec e p e ze fo e on on y np A p n n e een nd on y d n n nc n en e e ec o y c on en e fo nfe ence of e e

The fo o n theo e ze the p ope e of the e ne e Q 4 O M nde cond on 4 4 $n \to \infty$

$$\sqrt{pn}\sigma_0^{-1}\ p\ n\ 4\ \mu_{p,h} - \mu_p 4 \xrightarrow{d} N\ \ ,\ 4 \qquad 4$$

nd f the e o e

$$Bias\ \mu_{p,h} 4 \quad -\tfrac{1}{2}p^{-1}\sigma_K^2\ {}^2 f\ \nu_p 4 \quad o\ \ {}^2 4 \quad \text{nd} \qquad 4$$

$$Var\ \mu_{p,h} 4 \quad p^{-1}n^{-1}\sigma_0^2\ p\ n 4 \quad o\ n^{-1}\ 4 \qquad 4$$

By co p n the theo e fo nd the e ne e Q the e e p Q c no d on the n o ed p e e Q μ_p Q the co e pond n e fo e on epo ed n C en nd n 4 e no e th μ_p nd $\mu_{p,h}$ con e e Q μ_p the e of $\sqrt{n}$ o o e p ec y the e of $\sqrt{np}$. The e the e e Q ν_p nd $\nu_{p,h}$ con e e Q ν_p the e of $\sqrt{n}$ o o e p ec y the e of $\sqrt{n} f\ \nu_p 4$ the e e f the e p o y den y of Y_t

The econd p of the theo e con ey d e en Q y f e on of n the e on the e ne e Q doe no o e nce ed c on the econd o de of n^{-1} the econd o de e n e A the e e e the o n n n n e d Q no e nce e n e e n e e o The efo e fo the p po of e n the [redacted] the e ne o n co n e p od c e The nde y n e n the e f c the [redacted] e ec e y e n p e e The c n e e ed the cc e y tho o n The e on Q nonp e c e on of the e n p e e The c n e e e ed y the e p e e n

o d e no ed the e o e e en on y pp c e fo po n e on of [redacted] o con c n con dence n e nd e n the ypo the e on μ_p n the p e nce of d dependence the e ne o n c n p y n c n o e n e n $\sigma_0^2\ p\ n 4$ the pec den y e on pp o the

4. SIMULATION STUDY

n th c on e epo e f on dy the the e the pe fo nce of the nonp e c [redacted] e Q the n o ec e Q con o the o e c nd n n the p e o c on e ode o en fo e o o Y_t n e on e

n

e ne e[illegible] on of[illegible]e [illegible]pec[illegible] den[illegible]y of

p $n/$ $q)$ and $\gamma^2(q)$ $\max_{0\le j\le 2q-1} E\left(\sum_{l=[jp]+1}^{[(j+1)p]} X_l\right)^2$ en n eq y en n Yo oy 4 $\gamma^2(q) \le Cp$ App y eo e n Bo 4 fo α n q ence

$$P\{|F_n(\nu_p+\epsilon_n) - F(\nu_p+\epsilon_n)| > C_1\epsilon_n\}$$

$$\le 4\exp\left(-\frac{C_1^2\epsilon_n^2 q}{\sigma^2(q)}\right) \quad \{\quad \frac{}{C_1\epsilon_n}\}^{1/2} q\alpha\{n/(q)\} \qquad \text{A}\ 4$$

e e $\sigma^2(q)$ $p^{-2}\gamma^2(q)$ ϵ_n $C\epsilon_n$ o o

$$4\exp\left(-\frac{C_1^2\epsilon_n^2 q}{\sigma^2(q)}\right) \le 4\exp\{-C_2\epsilon_n q\} \qquad \text{A}\ 4$$

e e $C_2 >$ nce $n\epsilon_n^2 \to \infty$ e n $q\epsilon_n \to \infty$ e e n A 4 con e e q ze o e ponen y f On e cond e of A 4 e e e c α n p e

$$\{\quad \left(\frac{}{C_1\epsilon_n}\right)^{1/2} q\alpha\{n/(q)\} \le C\epsilon_n^{-1/2} q\rho^{[n^{1/2}\log^{-1}(n)/2]} \qquad \text{A}\ 4$$

con e e q ze o e ponen y f oo co p e e p oof of Le □

LEMMA nde e cond on 4 4 nd fo ny $\kappa >$

$$n^{-1}\sum(Y_t-\nu_p)\{I(Y_t\ge\nu_p) - I(Y_t\ge\nu_p)\} = o_p(n^{-3/4+\kappa})$$

OO Le W_t $(Y_t-\nu_p)\{I(Y_t\ge\nu_p) - I(Y_t\ge\nu_p)\}$ e e $E(W_t)$ No e $E(W_t)$ $-I_{t1}$ I_{t2} e e

$$I_{t1} \quad E\{(Y_t-\nu_p)I(\nu_p\le Y_t<\nu_p)I(\nu_p>\nu_p)\} \quad \text{nd}$$

$$I_{t2} \quad E\{(Y_t-\nu_p)I(\nu_p\le Y_t<\nu_p)I(\nu_p<\nu_p)\}.$$

e o e e I_{t1} I_{t11} I_{t12} nd I_{t2} I_{t21} I_{t22} e e fo $a\in$, / 4 nd $\eta >$

$$I_{t11} \quad E\{(Y_t-\nu_p)I(\nu_p\le Y_t<\nu_p)I(\nu_p\ge\nu_p\ n^{-a}\eta)\},$$

$$I_{t12} \quad E\{(Y_t-\nu_p)I(\nu_p\le Y_t<\nu_p)I(\nu_p<\nu_p<\nu_p\ n^{-a}\eta)\},$$

$$I_{t21} \quad E\{(Y_t-\nu_p)I(\nu_p> Y_t\ge\nu_p)I(\nu_p\le\nu_p-n^{-a}\eta)\} \quad \text{nd}$$

$$I_{t22} \quad E\{(Y_t-\nu_p)I(\nu_p> Y_t\ge\nu_p)I(\nu_p>\nu_p>\nu_p-n^{-a}\eta)\}.$$

App y n e C y z neq y fo k nd

$$|I_{tk1}| \le \sqrt{E(\nu_p-\nu_p)^2 P(|\nu_p-\nu_p|\ge n^{-a}\eta)}$$

en Le nd e f c $E\ \nu_p - \nu_p)^2 \ O\ n^{-1})$ p y

$$I_{tk1} \rightarrow \quad \text{e ponen y f} \qquad \text{A}$$

o e e I_{t12} e no e $|I_{t12}| \leq E\{\ Y_t - \nu_p) I\ \nu_p \leq Y_t < \nu_p \quad n^{-a}\eta)\}$. e n

$$I_{t12} \leq \int_{\nu_p}^{\nu_p + n^{-a}\eta} dy\ z - \nu_p) f\ z) dz \quad O\ n^{-2a})$$

n e e c y e pp o e c n o $I_{t22} \ O\ n^{-2a})$ e e nd A en y oo n $a \ - / \ \gamma$ e e $\gamma >$ y

$$E\ W_t) \quad o\ n^{-1+\kappa}) \qquad \text{A}$$

fo n y po e κ n n p e

$$E\Big[n^{-1} \sum\ Y_t - \nu_p)\{I\ Y_t \geq \mu_p) - I\ Y_t \geq \nu_p)\}\Big] \quad o\ n^{-1+\kappa}) \qquad \text{A}$$

e no con de $Var\ W_i)$ o $a \in \ , /)$

$$\begin{aligned}
E\ W_t^2) \quad & E\Big[\ Y_t - \nu_p)^2\{I\ Y_t \geq \nu_p) - \ I\ Y_t \geq \nu_p) I\ Y_t \geq \nu_p) \quad I\ Y_t \geq \nu_p)\}\Big] \\
& E\Big[\ Y_t - \nu_p)^2\{I\ \nu_p > Y_t \geq \nu_p) \quad I\ \nu_p > Y_t \geq \nu_p)\}\Big] \\
& E\Big[\ Y_t - \nu_p)^2 I\ \nu_p \leq Y_t < \nu_p)\{I\ \nu_p \geq \nu_p - n^{-a}\eta) \quad I\ \nu_p < \nu_{pn^{-a}}\eta \ a_\eta)\}\Big] \\
& E\Big[\ Y_t - \nu_p)^2 I\ \nu_p > Y_t \geq \nu_p)\{I\ \nu_p \geq \nu_p \quad n^{-a}\eta) \quad I\ \nu_p < \nu_p \quad n^{-a}\eta)\}\Big].
\end{aligned}$$

No e

$$\begin{aligned}
& E\{I\ \nu_p \leq Y_t < \nu_p) I\ \nu_p \leq \nu_p - n^{-a}\eta)\} \leq P\ |\nu_p - \nu_p| \geq n^{-a}\eta) \quad \text{nd} \\
& E\{I\ \nu_p > Y_t \geq \nu_p) I\ \nu_p > \nu_p \quad n^{-a}\eta)\} \leq P\ |\nu_p - \nu_p| \geq n^{-a}\eta)
\end{aligned}$$

con e e q ze o e ponen y f p ed y Le App y n e C y z neq y e

$$\begin{aligned}
& E\{\ |Y_t - \nu_p)^2 I\ \nu_p \leq Y_t < \nu_p) I\ \nu_p \leq \nu_p - n^{-a}\eta)\} \quad \text{nd} \\
& E\{\ |Y_t - \nu_p)^2 I\ \nu_p > Y_t \geq \nu_p) I\ \nu_p \geq \nu_p \quad n^{-a}\eta)\}
\end{aligned}$$

con e e q ze o e ponen y f e en pp y n e e od e A e

$$E\{ Y_t - \nu_p)^2 I \ \nu_p \le Y_t < \nu_p) I \ \nu_p \ge \nu_p - n^{-a}\eta)\} \quad O\ n^{-3a}) \quad \text{nd}$$
$$E\{ Y_t - \nu_p)^2 I \ \nu_p > Y_t \ge \nu_p) I \ \nu_p < \nu_p \quad n^{-a}\eta)\} \quad O\ n^{-3a})$$

n y e $E\ W_t^2)$ $o\ n^{-3/2+\kappa})$ nd A e n $Var\ W_t)$ $o\ n^{-3/2+\kappa})$ By y od fy n e o e de on fo $Var\ W_t)$ y e o n fo ny t_1, t_2 $Cov\ W_{t_1}, W_{t_2})$ $o\ n^{-3/2+\kappa})$ e efo e

$$Var\left[n^{-1}\sum_{i=1}^{n} Y_t - \nu_p)\{I\ Y_t \ge \mu_p) - I\ Y_t \ge \nu_p)\}\right] \quad o\ n^{-3/2+\kappa}) \qquad \text{A}$$

q e A e d y e e e e

LEMMA Le β $np)^{-1}\sum Y_t G_h\ \nu_p - Y_t)$ nd η $n\ )^{-1}\sum_{i=1}^{n} Y_t K_h\ \nu_p - Y_t)$ nde e cond on

$$Cov\left[\beta, \{p - S_h\ \nu_p)\}\{f\ \nu_p) - f\ \nu_p)\}\right] \quad o\ n^{-1}\ )$$
$$Cov\left[\beta,\ \eta - \eta)\{p - S_h\ \nu_p)\}\right] \quad o\ n^{-1}\ )$$
$$\text{c)}\ Cov\left[\{p - S_h\ \nu_p)\},\ \eta - \eta)\{p - S_h\ \nu_p)\}\right] \quad o\ n^{-1}\ )$$

OO e on y p e n e p oof of e p oof fo e e e De ne β $E\ \beta)$ Le $\beta - \beta$ $n^{-1}\sum \psi_1\ Y_t)$ $f\ \nu_p) - f\ \nu_p)$ $n^{-1}\sum \psi_2\ Y_t)$ O $^2)$ nd $p - F_h\ \nu_p)$ $n^{-1}\sum \psi_3\ Y_t)$ O $^2)$ fo e f nc on ψ_j j , nd $E\{\psi_j\ Y_t)\}$ o n nce $\psi_2\ Y_t)$ $K_h\ \nu_p - Y_t) - E\{K_h\ \nu_p - Y_t)\}$ nd $\psi_3\ Y_t)$ $G_h\ \nu_p - Y_t) - E\{G_h\ \nu_p - Y_t)\}$ n e pp o n B n ey p

$$\text{A} \quad |E\left[\ \beta - \beta)\{p - S_h\ \nu_p)\}\{f\ \nu_p) - f\ \nu_p)\}\right]$$
$$\le \quad n^{-2} \sum_{i\ge1, j\ge1, i+j\le n} |E\{\psi_1\ Y_1)\psi_2\ Y_i)\psi_3\ Y_{i+j})\}| \qquad O\ n^{-1}\ ^4 \quad n^{-2}\ ^2) \qquad \text{A}$$

e e nd c e e d e en pe on on e ee nd ce Le p δ q δ nd s^{-1} $- p^{-1} - q^{-1}$ fo e po e δ e D ydo neq y

$$|E\{\psi_1\ Y_1)\psi_2\ Y_i)\psi_3\ Y_{i+j})\}| \le \quad ||\psi\ Z_1)||_p||\psi_2\ Y_t)\psi_3\ Z_{i+j})||_q \alpha^{1/s}$$

e e o e p oy n e o

By [illegible] oo[illegible]n r [illegible] n^a fo $a \in$ [illegible] , [illegible] nd k' [illegible] n^c [illegible] $c \in$ [illegible] , $- a$ [illegible] e c n[illegible]o [illegible] e ef[illegible] nd [illegible]de of A [illegible] con e e[illegible] o [illegible] $n \to \infty$ ence

$$ {}_n \xrightarrow{p} \quad n \to \infty. \qquad \text{A} $$

[illegible] e efo e $S_{n,1}$ [illegible] $n^{-1/2} \sum_{j=1}^{r} W_{j,n}$ o_p [illegible]

By pp y n [illegible]e neq [illegible] y e[illegible] [illegible]ed n Yo oy[illegible] [illegible] nd[illegible]e con[illegible] c on of $W_{j,n}$ [illegible]

Le η $E\{p^{-1}Y_tK_h(Y_t-\nu_p)\}$ $p^{-1}\int(\nu_p-u)K(u)f(\nu_p-u)du$ $p^{-1}\nu_pf(\nu_p)$ $O(^2)$ n nd d de on fo α n q ence fo n nce o en n Bo e $Cov\{(np)^{-1}\sum Y_tK_h(\nu_p-Y_t),\nu_{p,h}-\nu_p\}$ $O(n^{-1})$ ence f A

$$E\{(np)^{-1}\sum Y_tK(\nu_p-Y_t)(\nu_{p,h}-\nu_p)\} \quad \eta E(\nu_{p,h}-\nu_p) \quad O(n^{-1})$$

$$-\tfrac{1}{2}p^{-1}\nu_pf'(\nu_p)^2\sigma_K^2 \quad o(^3) \quad O(n^{-1}) \qquad \text{A}$$

Co ne A A nd A

$$E(\mu_{p,h}) \quad \mu_p + \tfrac{1}{2}p^{-1}\sigma_K^2\,{}^2f(\nu_p) \quad o(^2) \quad O(n^{-1})$$

e e e en n

e no de e nce of $\mu_{p,h}$ Le A_1 $(np)^{-1}\sum_{t=1}^{n}\{Y_tG_h(\nu_p-Y_t)-Y_tK_h(\nu_p-Y_t)(\nu_{p,h}-\nu_p)\}$ e e d n o de e of e p n on A

en

$$Var(A_1) \quad Var\{(np)^{-1}\sum Y_tG_h(\nu_p-Y_t)\} \quad Var\{\eta(\nu_{p,h}-\nu_p)\}$$

$$- \quad Cov\{(np)^{-1}\sum Y_tG_h(\nu_p-Y_t),\eta(\nu_{p,h}-\nu_p)\}. \qquad \text{A}$$

e e

$$Var\{(np)^{-1}\sum Y_tG_h(\nu_p-Y_t)\}$$

$$n^{-1}p^{-2}\Big[Var\{Y_tG_h(\nu_p-Y_t)\} \quad \sum_{k=1}^{n-1}(-k/n)Cov\{Y_1G_h(\nu_p-Y_1),Y_{k+1}G_h(\nu_p-Y_{k+1})\}\Big].$$

Le c_K $\int_{-\infty}^{\infty}uK(u)du\int_{-\infty}^{u}K(v)dv$ y e o n

$$Var\{Y_tG_h(\nu_p-Y_t)\} \quad \int z^2G_h^2(\nu_p-z)f(z)dz-p^2\mu_p^2 \quad O(^2)$$

$$\int_{-\infty}^{\infty}K(v)dv\Big[\int_{-\infty}^{u}K(v)dv\{\int_{\nu_p}^{\infty}z^2f(z)dz \quad \int_{\nu_p-hv}^{\nu_p}z^2f(z)dz\}$$

$$\int_{u}^{\infty}K(v)dv\{\int_{\nu_p}^{\infty}z^2f(z)dz \quad \int_{\nu_p-hu}^{\nu_p}z^2f(z)dz\}-p^2\mu_p^2 \quad O(^2)$$

$$Var\{Y_tI(Y_t\geq\nu_p)\} - \quad \nu_p^2f(\nu_p)c_K \quad O(^2) \qquad \text{A}$$

on A nd Le e n

$$Var\{(np)^{-1}\sum Y_tG_h(\nu_p-Y_t)\} \quad p^{-2}Var\{\phi_1(\nu_p)\} - n^{-1}\quad \nu_p^2f(\nu_p)c_K \quad o\,n^{-1} \qquad \text{A}$$

e econd e on e nd de of A

$$Var\{\eta\ \nu_{p,h} - \nu_p)\} \quad \eta - \eta)\ \nu_{p,h} - \nu_p)\}$$

$$\eta^2 Var\ \nu_{p,h}) \quad \eta Cov\ \nu_{p,h},\ \eta - \eta)\ \nu_{p,h} - \nu_p)\} \quad Var\{\ \eta - \eta)\ \nu_{p,h} - \nu_p)\}.$$

y e o n y n e f c η $p^{-1}\nu_p f\ \nu_p)$ O $^2)$

$$\eta^2 Var\ \nu_{p,h}) \quad p^{-2}\nu_p^2 Var\{n^{-1}\sum_{t=1}^{n} I\ Y_t > \nu_p)\} - p^{-2}n^{-1}b\nu_p^2 f\ \nu_p) c_K \quad o\ n^{-1}b) \qquad A\ )$$

e neq y en n Yo oy) fo α n q ence

$$E\ \nu_{p,h} - \nu_p)^4 \le Cn^{-2} \quad \text{nd} \quad E\ \eta - \eta)^4 \quad O\ n^{-2}\ ^{-3})$$

App y n e C y z neq y nd Le

$$Var\{\ \eta - \eta)\ \nu_{p,h} - \nu_p)\} \quad O\ n^{-2}\ ^{-3/2}) \quad o\ n^{-1}\ ) \quad \text{nd} \qquad A\ )$$

$$Cov\{\eta\ \nu_{p,h} - \nu_p), p^{-1}\ \eta - \eta)\ \nu_{p,h} - \nu_p)\} \quad o\ n^{-1}\ ) \qquad A\ )$$

C ne A) A) nd A)

$$Var\{\eta\ \nu_{p,h} - \nu_p)\} \quad p^{-2}\nu_p^2 Var\{\phi_2\ \nu_p)\} - p^{-2}n^{-1}\ \nu_p^2 f\ \nu_p) c_K \quad o\ n^{-1}\ ) \qquad A\ )$$

Le e co nce e on e nd de of A

$$Cov\{\ np)^{-1}\sum_{t=1}^{n} Y_t G_h\ \nu_p - Y_t), \eta\ \nu_{p,h} - \nu_p)\}$$

$$Cov\{\ np)^{-1}\sum_{t=1}^{n} Y_t G_h\ \nu_p - Y_t), \eta f^{-1}\ \nu_p) n^{-1}\sum_{t=1}^{n} G_h\ \nu_p - Y_t)\} \quad o\ n^{-1}\ )$$

$$np^2)^{-1}\nu_p\Big[Cov\{Y_t G_h\ \nu_p - Y_t), G_h\ \nu_p - Y_t)\}$$

$$\sum_{k=1}^{n-1}\ - k/n) Cov\{Y_1 G_h\ \nu_p - Y_1), G_h\ \nu_p - Y_{k+1})\}\Big] \quad o\ n^{-1}\ )$$

nce $Cov\{Y_t G_h\ \nu_p - Y_t), G_h\ \nu_p - Y_t)\}$ p $- p)\nu_p -\ \nu_p f\ \nu_p)\ c_K$ o $)$

$$Cov\{\ np)^{-1}\sum_{t=1}^{n} Y_t G_h\ \nu_p - Y_t), \ np)^{-1}\sum_{i=1}^{n} Y_t K_h\ \nu_p - Y_t)\nu_{p,h} - \nu_p)\} \qquad A\ )$$

$$n^{-1}p^{-2}\nu_p Cov\{\phi_1\ \nu_p), \phi_2\ \nu_p)\} - n^{-1}p^{-2}\nu_p^2 f\ \nu_p)\ c_K \quad o\ )$$

e A A nd A o A e no e e econd o de e of $O(n^{-1})$ c nce o e e e nd e efo e

$$Var(\mu_p) = p^{-1} n^{-1} \sigma_0^2(p, n) = o(n^{-1}) \qquad \text{A}$$

e

e p o c n B y of $\nu_{p,h}$ c n e e ed f A y n e e oc n e od n e p oof of eo e

NC

A zne De en e M nd e D Ge e en e e of *M the tic Fin nce* **9**

Be n nd nc De ec n nd ode n dependence M n p nd p pe p e n ed n n e Me od n nce Confe ence C n e c d e d e ne Be n ey nd McNe A 4 nd e pec ed o f n po fo o of dependen c ed concep nd p c c n *ourn of B nking nd Fin nce*

enon C o e e nd C L edo 4 c o y Mode dden M o Mode nd c App c on *Bernoulli*

o e o C c e O nd L en 4 en y n y of e *ourn of e piric n nce*

o on 4 *t Risk* nd on Ne Yo Mc

Le d e e M L nd en nd oo zen 4 *E tre es nd Re ted Properties of R ndo Sequences nd Processes* p n e Be n

M y nd e D 4 Nonp e c e on nd den c on of non ne e e *Econo etric Theory* **11**

e D nd M 4 *St tistic An ysis of E tre e es ith pp c tions to insur nce n nce hydro gy nd other e d nd ed* Bo on B e

o n M 4 A cen eo nd e α n cond on *Proc N t Ac d Sc S A* **42**

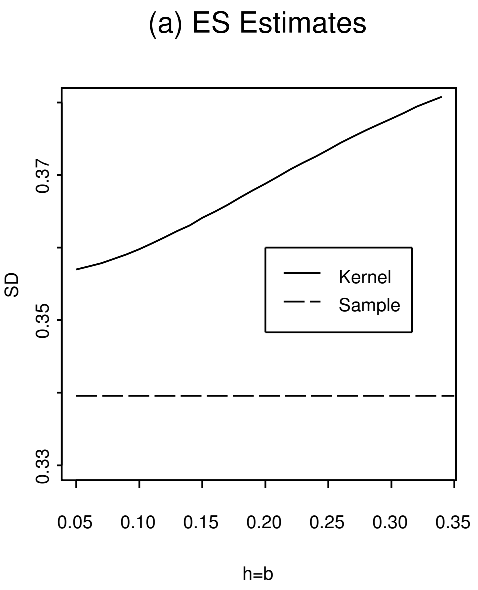
c e O 4 Nonp e c e on of cond on e pec ed o f C en e e eo

c e O 4 Nonp e c e on nd n y n y of e pec ed o f *M the tic Fin nce* **14**

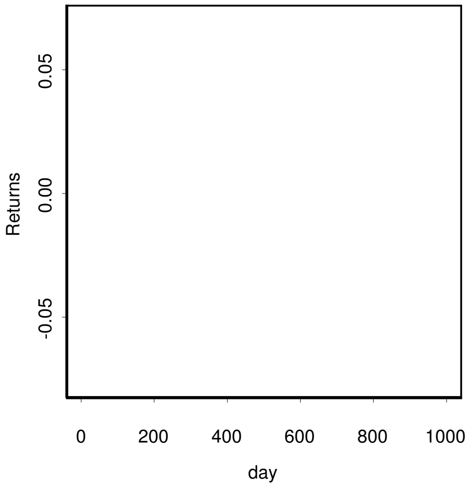
e e nd M on 4 e ne q n e e q *ourn of A eric n St tistic Associ tion* **5**

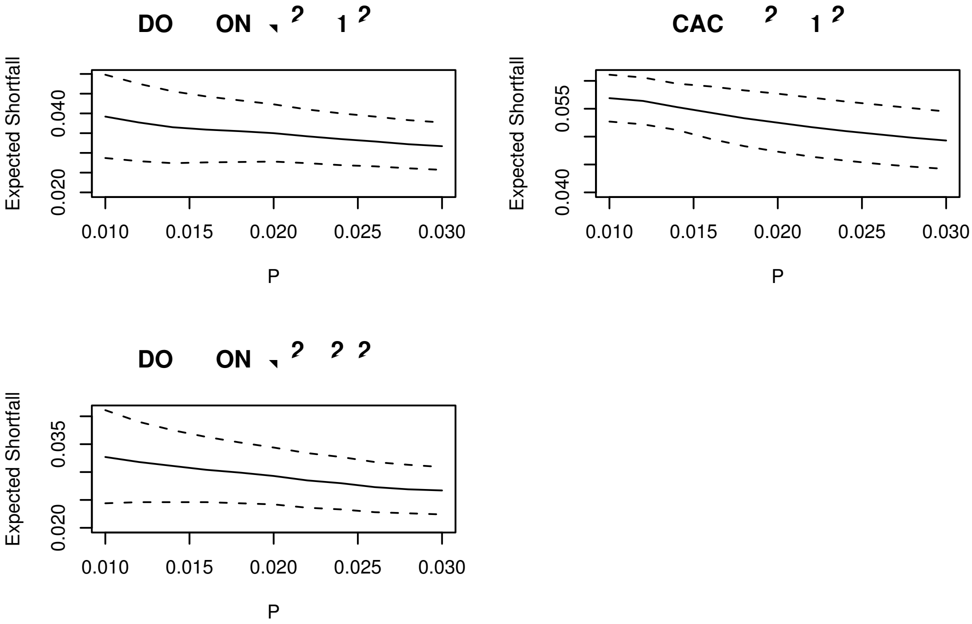
Yo oy 4 Mo en o nd fo on y n q ence *Pro ity Theory nd Re ted Fie ds* **52**

Y 4 e B d ep e n on of p e q n e fo q ence of on y n nd e *St tistics nd Pro ity Letters* **24**



(a) ES Estimates
SD
0.33
0.35
0.37
0.05
0.10
0.15
0.20
0.25
0.30
0.35
h=b
Kernel
Sample


0.05
0.00
-0.05
Returns
0
200
400
600
800
1000
day


Expected Shortfall
0.020
0.040
0.055
0.035
0.010
0.015
0.020
0.025
0.030
P

T . Esti tes for $\nu_{0.01}$ $\mu_{0.01}$ nd St nd rd Errors S.E.

	CAC			Do one		
Ye	$\nu_{0.01}$	μ_p		$\nu_{0.01}$	μ_p	